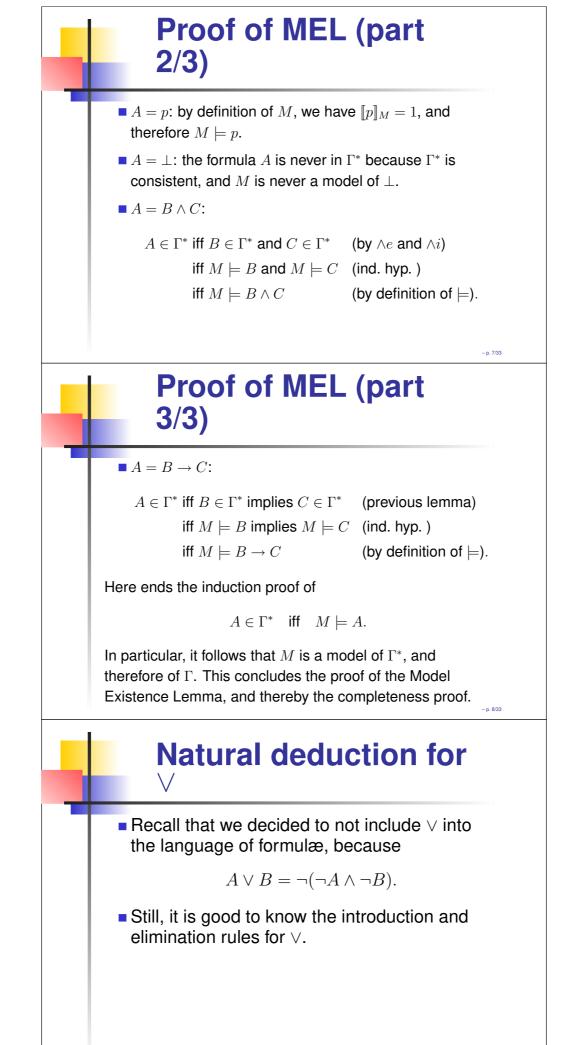
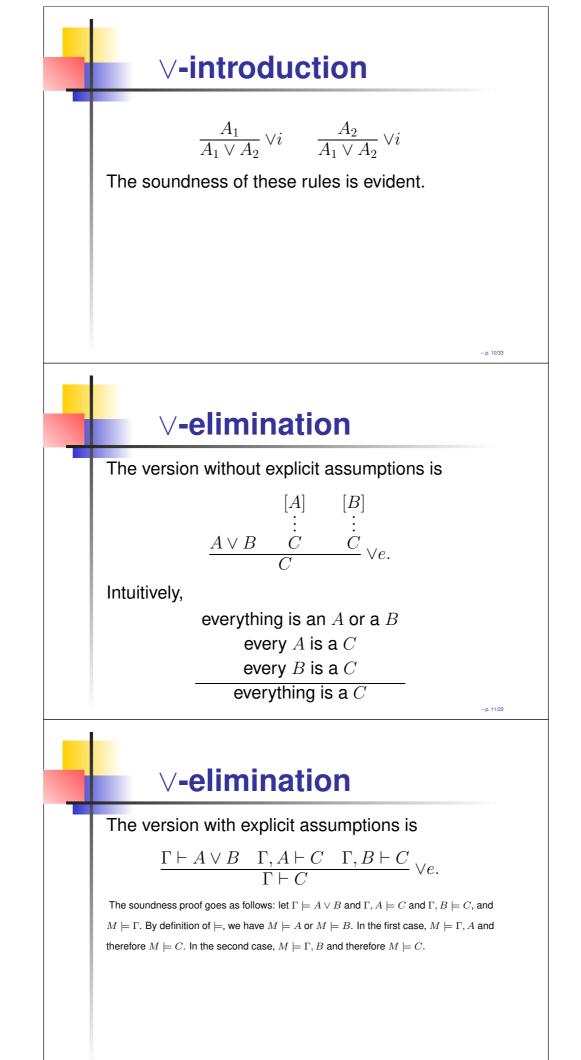
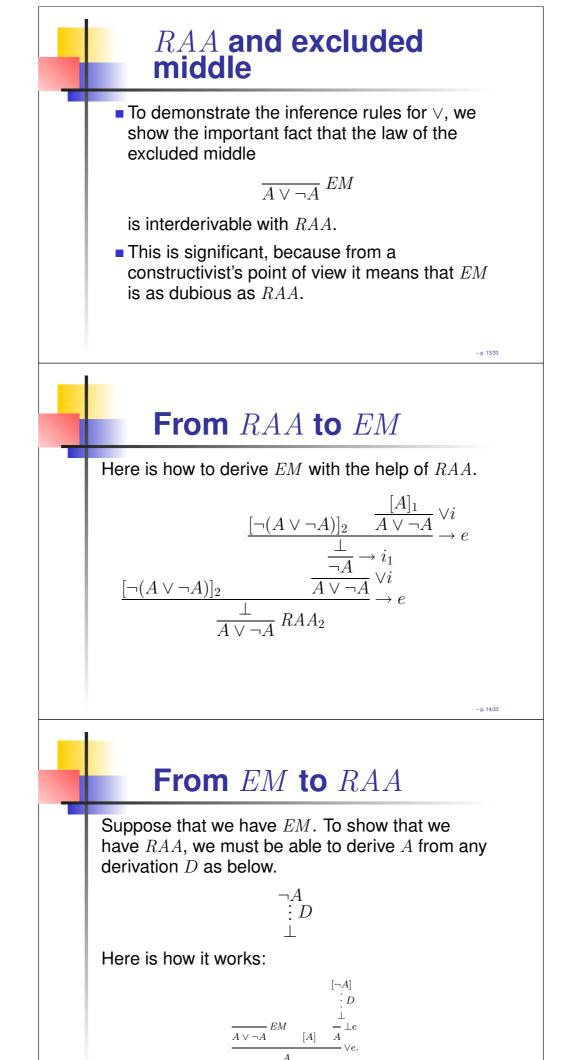
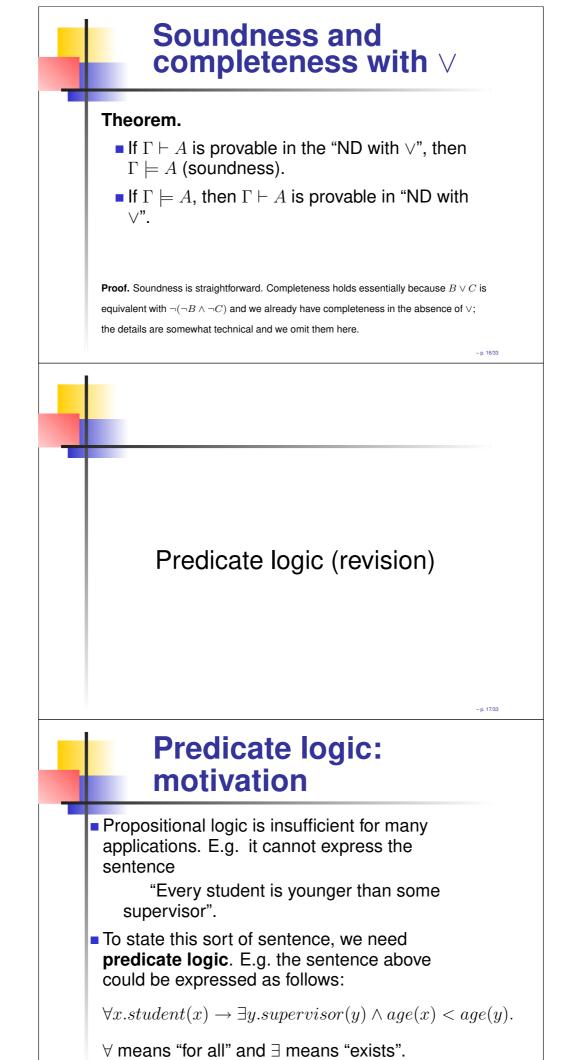


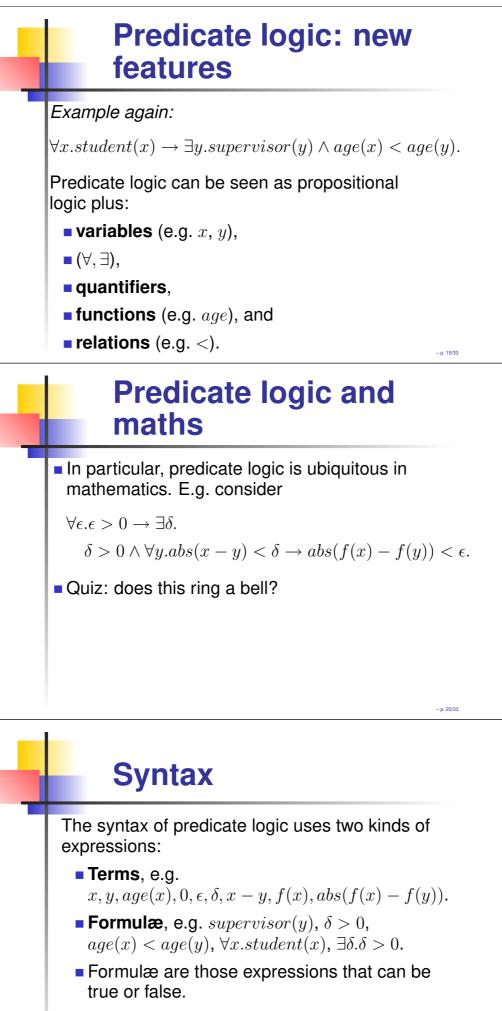
 $A \in \Gamma^*$ if and only if $M \models A$.



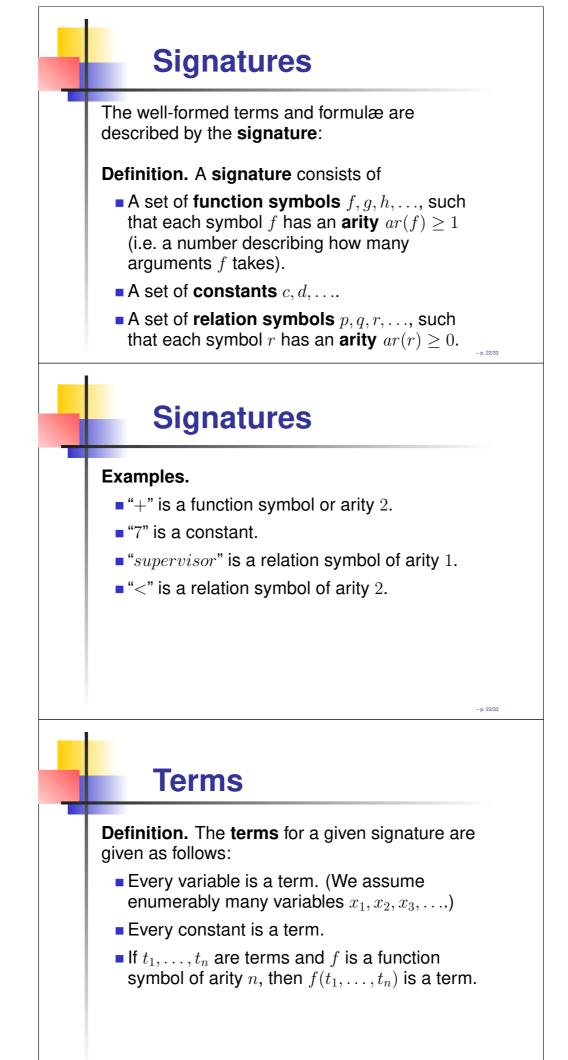








Terms stand for individuals of some universe.



Formulæ

Definition. The formulæ of predicate logic are given as follows:

- If t_1, \ldots, t_n are terms and p is a predicate symbol of arity n, then $p(t_1, \ldots, t_n)$ is a formula.
- If A and B are formulæ, then so are $(A \land B)$ and $(A \lor B)$ and $(A \to B)$;
- if A is a formula, then so is $(\neg A)$.
- \blacksquare \top and \bot are formula.
- If x is a variable and A is a formula, then $(\forall x.A)$ and $(\exists x.A)$ are formulæ.

– p. 25/33

- p. 26/33

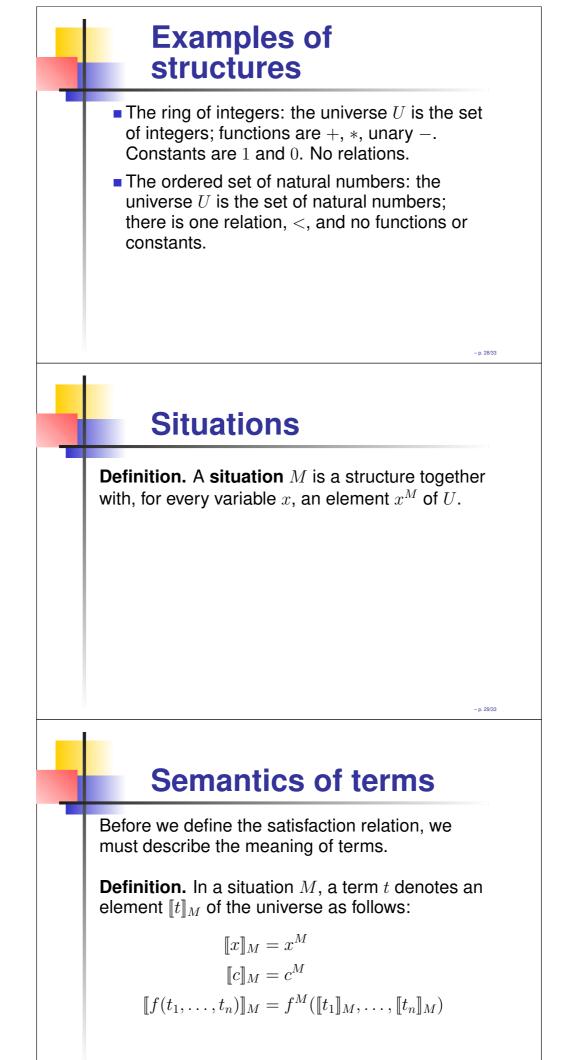
Semantics

- A situation for predicate logic is a pair consisting of a structure and a variable assignment.
- The structure describes the functions and relations corresponding to the the function symbols and relation symbols.
- The variable assignment sends each variable to an element of the **universe** on which the functions and relations are defined.

Structures

Definition. A **structure** M for a given signature consists of

- a non-empty set U called **universe**,
- for every constant c, an element of U,
- for every function symbol f of arity n, an n-ary function f^M , and
- for every relation symbol p of arity n, an n-ary relation p^M.



Semantics of formulæ

Definition. The satisfaction relation for predicate logic is defined as follows, where M[a/x] stands for the situation that is like M except that the variable x is interpreted as a.

$$\begin{split} M &\models p(t_1, \dots, t_n) \text{ if } (\llbracket t_1 \rrbracket_M, \dots, \llbracket t_n \rrbracket_M) \in p^M \\ M &\models \forall x.A \text{ if for all } a \in U \text{ it holds that } M[a/x] \models A \\ M &\models \exists x.A \text{ if there exists an } a \in U \text{ such that } M[a/x] \models A \\ M &\models A \land B \text{ if } M \models A \text{ and } M \models B \\ M &\models A \lor B \text{ if } M \models A \text{ or } M \models B \\ M &\models A \to B \text{ if } M \models A \text{ implies } M \models B \\ M &\models \neg A \text{ if } M \not\models A \\ M &\models \neg \text{ never} \\ M &\models \top \text{ never} \end{split}$$

Predicate logic vs. propositional logic

– p. 31/33

- p. 32/33

By definition of the semantics, for a nullary predicate symbol p we have

$$M \models p() \text{ if } () \in p^M$$

- Such a p has only two possible behaviours: $M \models p()$ or $M \not\models p()$.
- So nullary relation symbols take over the rôle of the propositional atoms.
- Thus propositional logic can be seen as the simplified case of predicate logic where all predicate symbols are nullary.

Validity, satisfiability, semantic entailment

The definitions of validity, satisfiability, and semantic entailment for predicate logic look exactly the same as for propositional logic.